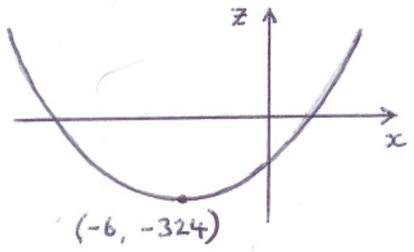
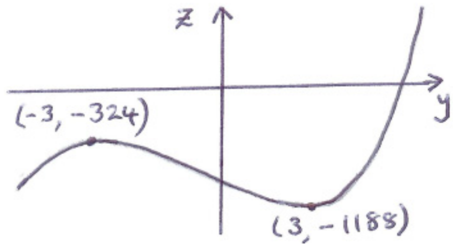
1 (i)	Distance is $\dfrac{2(-2)-(-7)+2(7)-11}{\sqrt{2^2+1^2+2^2}}$ $=2$	M1 A1 A1 **3**	Formula, or other complete method Numerical expression for distance
(ii)	$\begin{pmatrix}1\\4\\7\end{pmatrix}\times\begin{pmatrix}2\\-1\\2\end{pmatrix}=\begin{pmatrix}15\\12\\-9\end{pmatrix}$ Equation of AD is $\mathbf{r}=\begin{pmatrix}2\\-1\\3\end{pmatrix}+\lambda\begin{pmatrix}5\\4\\-3\end{pmatrix}$	M1 A1 A1 ft **3**	Vector product of normals, or finding a point on AD, e.g. $(0,-2.6,4.2)$, $(3.25,0,2.25)$, $(7,3,0)$ Correct direction Accept any form
(iii)	$\overrightarrow{CA}\times\mathbf{d}=\begin{pmatrix}-5\\-6\\2\end{pmatrix}\times\begin{pmatrix}5\\4\\-3\end{pmatrix}=\begin{pmatrix}10\\-5\\10\end{pmatrix}$ Distance is $\dfrac{\lvert\overrightarrow{CA}\times\mathbf{d}\rvert}{\lvert\mathbf{d}\rvert}=\dfrac{\sqrt{10^2+5^2+10^2}}{\sqrt{5^2+4^2+3^2}}=\dfrac{\sqrt{225}}{\sqrt{50}}$ $=\sqrt{4.5}=\dfrac{3}{\sqrt{2}}=\dfrac{3\sqrt{2}}{2}\approx 2.12$	M1 A2 ft M1 M1 A1 **6**	Appropriate vector product Give A1 if just one error Formula for distance Finding magnitude *Both dependent on first M1*
(iv)	$\mathbf{d}\times\overrightarrow{BC}=\begin{pmatrix}5\\4\\-3\end{pmatrix}\times\begin{pmatrix}9\\12\\-6\end{pmatrix}=\begin{pmatrix}12\\3\\24\end{pmatrix}=3\begin{pmatrix}4\\1\\8\end{pmatrix}$ Distance is $\dfrac{\begin{pmatrix}-4\\-6\\4\end{pmatrix}.\begin{pmatrix}4\\1\\8\end{pmatrix}}{\sqrt{4^2+1^2+8^2}}=\dfrac{10}{9}$	M1 A1 ft M1 A1 ft M1 A1 **6**	Vector product of directions Appropriate scalar product Evaluation of scalar product For denominator
(v)	$V=\frac{1}{6}(\overrightarrow{AB}\times\overrightarrow{AC}).\overrightarrow{AD}=\frac{1}{6}\left[\begin{pmatrix}-4\\-6\\4\end{pmatrix}\times\begin{pmatrix}5\\6\\-2\end{pmatrix}\right].\lambda\begin{pmatrix}5\\4\\-3\end{pmatrix}$ $=\frac{1}{6}\lambda\begin{pmatrix}-12\\12\\6\end{pmatrix}.\begin{pmatrix}5\\4\\-3\end{pmatrix}=-5\lambda$ $V=\pm 20\ \Rightarrow\ \lambda=\pm 4$ D is $(22,15,-9)$ or $(-18,-17,15)$	M1 M1 A1 ft M1 A1A1 **6**	Appropriate scalar triple product Evaluation of scalar triple product or $-2a+2b+c+3$ (simplified) for $\mathrm{D}(a,b,c)$ Obtain a value of λ, or one of a, b, c

Alternative methods for Question 1

1 (ii)	Eliminating x $3y+4z=9$ $x=7-\frac{5}{3}t,\ \ y=3-\frac{4}{3}t,\ \ z=t$	M1 A1A1 **3**	Eliminating one of x, y, z or $3x+5z=21$ or $4x-5y=13$
1 (iii)	$\left[\begin{pmatrix}2\\-1\\3\end{pmatrix}+\lambda\begin{pmatrix}5\\4\\-3\end{pmatrix}-\begin{pmatrix}7\\5\\1\end{pmatrix}\right].\begin{pmatrix}5\\4\\-3\end{pmatrix}=0$ $25\lambda-25+16\lambda-24+9\lambda-6=0$ $\lambda=1.1$, F is $(7.5, 3.4, -0.3)$ $\text{CF}=\sqrt{(0.5)^2+(-1.6)^2+(-1.3)^2}$ $=\sqrt{4.5}$	M1 A1 ft A1 ft M1 M1 A1 **6**	Appropriate scalar product Obtaining a value for λ Finding magnitude
1 (iv)	$\left[\begin{pmatrix}-2\\-7\\7\end{pmatrix}+\mu\begin{pmatrix}9\\12\\-6\end{pmatrix}-\begin{pmatrix}2\\-1\\3\end{pmatrix}-\lambda\begin{pmatrix}5\\4\\-3\end{pmatrix}\right].\begin{pmatrix}5\\4\\-3\end{pmatrix}=0$ and $\begin{pmatrix}-5\lambda+9\mu-4\\-4\lambda+12\mu-6\\3\lambda-6\mu+4\end{pmatrix}.\begin{pmatrix}9\\12\\-6\end{pmatrix}=0$ $\lambda=\frac{4}{81},\ \ \mu=\frac{128}{243}$ Distance is $\sqrt{\left(\frac{40}{81}\right)^2+\left(\frac{10}{81}\right)^2+\left(\frac{80}{81}\right)^2}$ $=\frac{10}{9}$	M1 A1 ft A1 ft M1 M1 A1 **6**	Two appropriate scalar products Obtaining values for λ and μ Obtaining distance

2 (i)	When $y=-3$, $z=3x^2+36x-216$ 	B1 B1 B1	 Correct shape (parabola) and position For $(-6,-324)$
	When $x=-6$, $z=8y^3-216y-756$ 	B1 B1 B1 B1 **7**	 Correct shape and position For $(-3,-324)$ For $(3,-1188)$ *If B0B0 then give B1 for* $x=\pm 3$
(ii)	$(-6,-3,-324)$ is a SP on both sections; hence it is a SP on S Saddle point	B1 B1 **2**	
(iii)	$\frac{\partial z}{\partial x}=-12xy-30x+36,\quad \frac{\partial z}{\partial y}=24y^2-6x^2$ At a SP, $\frac{\partial z}{\partial x}=\frac{\partial z}{\partial y}=0$ $24y^2-6x^2=0 \Rightarrow y=\pm\frac{1}{2}x$ $y=\frac{1}{2}x \Rightarrow -6x^2-30x+36=0$ $\Rightarrow x=-6,\ 1;$ SPs are $(-6,-3,-324)$ $(1,\ 0.5,\ 19)$ $y=-\frac{1}{2}x \Rightarrow 6x^2-30x+36=0$ $\Rightarrow x=2,\ 3;$ SPs are $(2,-1,\ 28)$ $(3,-1.5,\ 27)$	B1B1 M1 M1 A1 M1 A1 A1 **8**	
(iv)	$\frac{\partial z}{\partial x}=120$ and $\frac{\partial z}{\partial y}=0$ $y=\frac{1}{2}x \Rightarrow -6x^2-30x-84=0;\ D=30^2-4\times 6\times 84$ $D\ (=-1116)<0$; so this has no roots $y=-\frac{1}{2}x \Rightarrow 6x^2-30x-84=0 \Rightarrow x=7,-2$ When $x=7,\ y=-3.5,\ z=203;$ so $k=637$ When $x=-2,\ y=1,\ z=-148;$ so $k=-92$	M1 M1 A1 M1 M1 A1 A1 **7**	(Allow M1 for $\frac{\partial z}{\partial x}=-120$) Obtaining at least one value of x Obtaining a value of k

3(a)(i)	$\frac{dy}{dx} = \frac{1}{2}e^{\frac{1}{2}x} - \frac{1}{2}e^{-\frac{1}{2}x}$ $\left(\frac{dy}{dx}\right)^2 = \frac{1}{4}e^x - \frac{1}{2} + \frac{1}{4}e^{-x}$ $1 + \left(\frac{dy}{dx}\right)^2 = \frac{1}{4}e^x + \frac{1}{2} + \frac{1}{4}e^{-x} = (\frac{1}{2}e^{\frac{1}{2}x} + \frac{1}{2}e^{-\frac{1}{2}x})^2$	B1 M1 A1 (ag) **3**	 Correct completion
(ii)	Length is $\int_0^{\ln a} (\frac{1}{2}e^{\frac{1}{2}x} + \frac{1}{2}e^{-\frac{1}{2}x})\,dx$ $= \left[e^{\frac{1}{2}x} - e^{-\frac{1}{2}x} \right]_0^{\ln a}$ $= \left(\sqrt{a} - \frac{1}{\sqrt{a}} \right) - (1-1) = \frac{a-1}{\sqrt{a}}$	M1 A1 A1 (ag) **3**	For $\int (\frac{1}{2}e^{\frac{1}{2}x} + \frac{1}{2}e^{-\frac{1}{2}x})\,dx$ For $e^{\frac{1}{2}x} - e^{-\frac{1}{2}x}$ Correctly shown
(iii)	Curved surface area is $\int 2\pi y\,ds$ $= \int_0^{\ln a} 2\pi(e^{\frac{1}{2}x} + e^{-\frac{1}{2}x})(\frac{1}{2}e^{\frac{1}{2}x} + \frac{1}{2}e^{-\frac{1}{2}x})\,dx$ $= \pi \int_0^{\ln a} (e^x + 2 + e^{-x})\,dx$ $= \pi \left[e^x + 2x - e^{-x} \right]_0^{\ln a}$ $= \pi \left(a + 2\ln a - \frac{1}{a} \right)$	M1 A1 M1 A1 A1 **5**	For $\int y\,ds$ Correct integral form *including limits* Obtaining integrable expression For $e^x + 2x - e^{-x}$
(b)(i)	$\frac{dy}{dx} = \frac{\cos\theta}{-2\sin\theta}$ Gradient of normal is $\frac{2\sin\theta}{\cos\theta}$ $(= 2\tan\theta)$ Normal is $y - \sin\theta = 2\tan\theta(x - 2\cos\theta)$ $y = 2x\tan\theta - 3\sin\theta$	B1 M1 A1 (ag) **3**	 Correctly shown
(ii)	Differentiating partially w.r.t. θ $0 = 2x\sec^2\theta - 3\cos\theta$ $x = \frac{3}{2}\cos^3\theta$ $y = 3\cos^3\theta\tan\theta - 3\sin\theta$ $= 3\sin\theta(\cos^2\theta - 1) = -3\sin^3\theta$ $(2x)^{\frac{2}{3}} + y^{\frac{2}{3}} = (3\cos^3\theta)^{\frac{2}{3}} + (-3\sin^3\theta)^{\frac{2}{3}}$ $= 3^{\frac{2}{3}}(\cos^2\theta + \sin^2\theta) = 3^{\frac{2}{3}}$	M1 A1 M1 A1 M1 A1 (ag) **6**	 Obtaining an expression for y Any correct form Using $1 - \cos^2\theta = \sin^2\theta$ Correctly shown
(iii) *(A)* *(B)*	$(2, 0)$ has $\theta = 0$ Centre of curvature is $(\frac{3}{2},\ 0)$ $\rho = \frac{1}{2}$ $(0, 1)$ has $\theta = \frac{1}{2}\pi$ Centre of curvature is $(0,\ -3)$ $\rho = 4$	M1 A1 M1 A1 **4**	Using param eqn with $\theta = 0$ *(or other method for ρ or cc)* Using param eqn with $\theta = \frac{1}{2}\pi$ *(or other method for ρ or cc)*

4 (i)	<table><tr><th></th><th>1</th><th>3</th><th>4</th><th>5</th><th>9</th></tr><tr><td>1</td><td>1</td><td>3</td><td>4</td><td>5</td><td>9</td></tr><tr><td>3</td><td>3</td><td>9</td><td>1</td><td>4</td><td>5</td></tr><tr><td>4</td><td>4</td><td>1</td><td>5</td><td>9</td><td>3</td></tr><tr><td>5</td><td>5</td><td>4</td><td>9</td><td>3</td><td>1</td></tr><tr><td>9</td><td>9</td><td>5</td><td>3</td><td>1</td><td>4</td></tr></table>	B2	Give B1 if not more than 4 errors
	Composition table shows closure Identity is 1	B1 B1	*Dependent on B2 for table*
	<table><tr><th>Element</th><th>1</th><th>3</th><th>4</th><th>5</th><th>9</th></tr><tr><td>Inverse</td><td>1</td><td>4</td><td>3</td><td>9</td><td>5</td></tr></table>So every element has an inverse	B2 **6**	Give B1 for 3 correct
(ii)	Since 5 is prime, a group of order 5 must be cyclic Two cyclic groups of the same order must be isomorphic	B1 B1 B1 **3**	
(iii)	<table><tr><th>*H*</th><th>1</th><th>$e^{\frac{2}{5}\pi j}$</th><th>$e^{\frac{4}{5}\pi j}$</th><th>$e^{\frac{6}{5}\pi j}$</th><th>$e^{\frac{8}{5}\pi j}$</th></tr><tr><td>*G*</td><td>1</td><td>3</td><td>9</td><td>5</td><td>4</td></tr><tr><td>*or*</td><td>1</td><td>4</td><td>5</td><td>9</td><td>3</td></tr><tr><td>*or*</td><td>1</td><td>5</td><td>3</td><td>4</td><td>9</td></tr><tr><td>*or*</td><td>1</td><td>9</td><td>4</td><td>3</td><td>5</td></tr></table>	B1 B2 **3**	For $1 \leftrightarrow 1$ For non-identity elements
(iv)	Identity is $(1,\ 1)$ Inverse of $(9,\ 3)$ is $(5,\ 4)$	B1 B1 **2**	
(v)	$(x,\ y)^5 = (x^5,\ y^5)$ Since *G* has order 5, $x^5 = 1$ and $y^5 = 1$ Hence $(x,\ y)^5 = (1,\ 1)$	M1 M1 A1 (ag) **3**	
(vi)	Order of $(x,\ y)$ is a factor of 5 (so must be 1 or 5) Only identity $(1,\ 1)$ can have order 1 Hence all other elements have order 5	M1 B1 A1 (ag) **3**	
(vii)	$\{(1,\ 1),\ (9,\ 3),\ (4,\ 9),\ (3,\ 5),\ (5,\ 4)\}$	B2 **2**	Give B1 ft for 5 elements including $(1,\ 1),\ (9,\ 3),\ (5,\ 4)$
(viii)	An element of order 5 generates a subgroup, and so can be in only one subgroup of order 5 Number is $24 \div 4 = 6$	M1 A1 **2**	*Or* for $24 \div 4$ *Or* listing at least 2 other subgroups *Give B1 for unsupported answer 6*

Pre-multiplication by transition matrix

5 (i)	$\mathbf{P}=\begin{pmatrix}1 & 0.07 & 0 & 0\\ 0 & 0.8 & 0.15 & 0\\ 0 & 0.13 & 0.75 & 0\\ 0 & 0 & 0.1 & 1\end{pmatrix}$	B2 **2**	*Allow tolerance of* ± 0.0001 *in probabilities throughout this question* Give B1 for two columns correct
(ii)	If system enters an absorbing state, it remains in that state A and D are absorbing states	B1 B1 **2**	
(iii)	$\mathbf{P}^9\begin{pmatrix}0\\0.4\\0.6\\0\end{pmatrix}=\begin{pmatrix}0.2236\\0.2505\\0.1998\\0.3261\end{pmatrix}$ Prob(owned by B) = 0.2505	M1M1 A1 **3**	For $\mathbf{P}^9$ (or $\mathbf{P}^{10}$) and $\begin{pmatrix}0\\0.4\\0.6\\0\end{pmatrix}$
(iv)	$\mathbf{P}^4=\begin{pmatrix}1 & \dots & \dots & \dots\\ \dots & 0.4818 & \dots & \dots\\ \dots & \dots & 0.3856 & \dots\\ \dots & \dots & \dots & 1\end{pmatrix}$ $0.2236+0.2505\times0.4818+0.1998\times0.3856+0.3261$ $= 0.7474$	M1 M1 A1 **3**	Using diagonal elements from $\mathbf{P}^4$ Using probabilities for 10th day
(v)	$\begin{pmatrix}1 & 0 & 0 & 1\end{pmatrix}\mathbf{P}^n\begin{pmatrix}0\\0.4\\0.6\\0\end{pmatrix}$ $=(\,0.8971\,)$ when $n=26$ $=(\,0.9057\,)$ when $n=27$ i.e. on the 28th day	M1 M1 A1 ft A1 **4**	Considering $\mathbf{P}^n$ for some $n>20$ Evaluating Prob(A or D) for some values of n Identifying $n=26$ or $n=27$ (*Implies M1M1*)
(vi)	$\mathbf{P}^n \to \begin{pmatrix}1 & \mathbf{0.5738} & \mathbf{0.3443} & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & \mathbf{0.4262} & \mathbf{0.6557} & 1\end{pmatrix}=\mathbf{Q}$	B2 **2**	Give B1 for two bold elements correct (to 3 dp)
(vii)	$\mathbf{Q}\begin{pmatrix}0\\0.4\\0.6\\0\end{pmatrix}=\begin{pmatrix}0.4361\\0\\0\\0.5639\end{pmatrix}$ Prob(eventually owned by A) = 0.4361	M1M1 A1 **3**	Using $\mathbf{Q}$ and $\begin{pmatrix}0\\0.4\\0.6\\0\end{pmatrix}$
(viii)	$\mathbf{Q}\begin{pmatrix}0\\p\\q\\0\end{pmatrix}=\begin{pmatrix}0.5\\0\\0\\0.5\end{pmatrix}$ (where $q=1-p$) $0.5738p+0.3443q=0.5$ $p=0.6786,\quad q=0.3214$	M1M1 A1 ft M1 A1 **5**	For LHS and RHS Or $0.4262p+0.6557q=0.5$ Solving to obtain a value of p Allow 0.678 - 0.679, 0.321 – 0.322